NCERT Class 12 Mathematics Solutions

Chapter 1 - Relations and Functions

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Exercise 1.1

1. Determine whether each of the following relations are reflexive, symmetric, and transitive:

(i) *Relation* R *in the set* $A = \{1, 2, 3, \ldots, 13, 14\}$ *defined as:*

$R = \{(x, y) : 3x - y = 0\}$

Solution :

Relation R *in the set* $A = \{1, 2, 3, \ldots, 13, 14\}$ *defined as:*

$R = \{(x, y) : 3x - y = 0\}$, *i.e.,* $y = 3x$.

- ***Reflexive***: *For a relation to be reflexive,* (x, x) *must be in* R *for all* $x \in A$. *Here,* $y = 3x$, *so* $x \neq 3x$, *unless* $x = 0$, *which is not in the set* A. *Thus,* R *is* ***not reflexive***.

- ***Symmetric***: *For* R *to be symmetric, if* $(x, y) \in R$, *then* (y, x) *should also be in* R. *If* $y = 3x$, *then for symmetry,* $x = 3y$, *which contradicts* $y = 3x$. *Therefore,* R *is* ***not symmetric***.

- ***Transitive***: *For transitivity, if* $(x, y) \in R$ *and* $(y, z) \in R$, *then* (x, z) *should be in* R. *Since* $y = 3x$ *and* $z = 3y$, *we have* $z = 9x$, *which does not satisfy the condition* $z = 3x$. *Thus,* R *is* ***not transitive***.

(ii) *Relation* R *in the set* $\mathbf{N}$*(natural numbers) defined as:*

$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$.

Solution :

Relation R *in the set* $\mathbf{N}$ *(natural numbers) defined as:*

$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$.

- ***Reflexive***: *To check if the relation is reflexive, we need* $x = y$, *but the relation defines* $y = x + 5$, *so no element is related to itself. Hence,* R *is* ***not reflexive***.

- ***Symmetric***: *For symmetry, if* $(x, y) \in R$*, then* $(y, x) \in R$ *should also hold. Since* $y = x + 5$*, the reverse* $x = y + 5$ *is not true for natural numbers. Thus,* R *is* ***not symmetric***.

- ***Transitive***: *For transitivity, if* $(x, y) \in R$ *and* $(y, z) \in R$*, then* (x, z) *should be in* R*. But since* $y = x+5$ *and* $z = y+5$*,* $z = x+10$*, which is not of the form* $z = x + 5$*. Thus,* R *is* ***not transitive***.

(iii) Relation R *in the set* $A = \{1, 2, 3, 4, 5, 6\}$ *defined as:*

$R = \{(x, y) : y \text{ is divisible by } x\}$.

Solution :

Relation R *in the set* $A = \{1, 2, 3, 4, 5, 6\}$ *defined as:*

$R = \{(x, y) : y \text{ is divisible by } x\}$.

- ***Reflexive***: *For every* $x \in A$*,* y *should be divisible by* x*, and since* x *is divisible by itself,* R *is* ***reflexive***.

- ***Symmetric***: *If* $(x, y) \in R$*, then* y *is divisible by* x*. For symmetry,* x *must also be divisible by* y*, which is not necessarily true. Thus,* R *is* ***not symmetric***.

- ***Transitive***: *If* y *is divisible by* x*, and* z *is divisible by* y*, then* z *must also be divisible by* x*. Hence,* R *is* ***transitive***.

(iv) Relation R *in the set* ***Z*** *(integers) defined as:*

$R = \{(x, y) : x - y \text{ is an integer}\}$.

Solution :

Relation R *in the set* ***Z*** *(integers) defined as:*

$R = \{(x, y) : x - y \text{ is an integer}\}$.

- ***Reflexive***: *Since* $x - x = 0$, *which is an integer,* R *is* ***reflexive***.
- ***Symmetric***: *If* $x - y$ *is an integer, then* $y - x$ *is also an integer (just the negative of the same integer), so* R *is* ***symmetric***.
- ***Transitive***: *If* $x - y$ *and* $y - z$ *are integers, then* $x - z$ *is also an integer (since the integers are closed under addition), so* R *is* ***transitive***.

(v) Relation R in the set of human beings in a town at a particular time:

(a) $R = \{(x, y) : x \text{ and } y \text{ work at the same place}\}$.

Solution :

- ***Reflexive***: *A person works at the same place as themselves, so it is* ***reflexive***.
- ***Symmetric***: *If* x *works at the same place as* y, *then* y *works at the same place as* x, *so it is* ***symmetric***.
- ***Transitive***: *If* x *works at the same place as* y *and* y *works at the same place as* z, *then* x *works at the same place as* z, *so it is* ***transitive***.

(b) $R = \{(x, y) : x \text{ and } y \text{ live in the same locality}\}$.

Solution :

- *This relation is similar to (a), so it is* ***reflexive***, ***symmetric***, *and* ***transitive***.

(c) $R = \{(x, y) : x \text{ is exactly 7 cm taller than } y\}$.

Solution :

- ***Reflexive***: *A person cannot be taller than themselves, so R is* ***not reflexive***.
- ***Symmetric***: *If x is 7 cm taller than y, y cannot be 7 cm taller than x, so it is* ***not symmetric***.
- ***Transitive***: *If x is 7 cm taller than y and y is 7 cm taller than z, then x is 14 cm taller than z, which does not satisfy the condition. Thus, R is* ***not transitive***.

(d) $R = \{(x, y) : x \text{ is the wife of } y\}$.

Solution :

- ***Reflexive***: *A person cannot be their own spouse, so it is* ***not reflexive***.
- ***Symmetric***: *If x is the wife of y, then y cannot be the wife of x, so it is* ***not symmetric***.
- ***Transitive***: *If x is the wife of y and y is the wife of z, this is not possible, so it is* ***not transitive***.

(e) $R = \{(x, y) : x \text{ is the father of } y\}$.

Solution :

- ***Reflexive***: *A person cannot be their own father, so it is* ***not reflexive***.
- ***Symmetric***: *If x is the father of y, then y cannot be the father of x, so it is* ***not symmetric***.
- ***Transitive***: *If x is the father of y and y is the father of z, then x is the grandfather of z, not the father, so it is* ***not transitive***.

*2. Show that the relation R in the set **R** of real numbers, defined as*

$$R = \{(a, b) : a \leq b^2\}$$

is neither reflexive, nor symmetric, nor transitive.

Solution :

To determine whether the relation $R = \{(a, b) : a \leq b^2\}$ is reflexive, symmetric, or transitive, we analyze each property separately.

Reflexivity:

For a relation to be reflexive, it must satisfy $(a, a) \in R$ for all $a \in \mathbf{R}$. This means that for every real number a, we must have:

$$a \leq a^2$$

Let's consider different values of a:

- *If $a = 0$, then $0 \leq 0^2$, which is true.*
- *If $a = 1$, then $1 \leq 1^2$, which is true.*
- *If $a = -1$, then $-1 \leq (-1)^2 = 1$, which is true.*
- *If $a = 0.5$, then $0.5 \leq (0.5)^2 = 0.25$, which is **false**.*

*Since $a \leq a^2$ does not hold for all $a \in \mathbf{R}$, the relation is **not reflexive**.*

Symmetry:

For a relation to be symmetric, it must satisfy $(a, b) \in R \implies (b, a) \in R$. This means if $a \leq b^2$, then we should also have $b \leq a^2$.

Consider the following counterexample:

- *Let $a = 0$ and $b = 1$.*

- $0 \leq 1^2 = 1$, *so* $(0, 1) \in R$.
- *But* $1 \leq 0^2 = 0$ *is* ***false***, *so* $(1, 0) \notin R$.

Since $(a, b) \in R$ *does not imply* $(b, a) \in R$ *for all* $a, b \in \mathbf{R}$, *the relation is* ***not symmetric***.

Transitivity:

For a relation to be transitive, it must satisfy $(a, b) \in R$ *and* $(b, c) \in R$ *implies* $(a, c) \in R$. *This means if* $a \leq b^2$ *and* $b \leq c^2$, *then* $a \leq c^2$ *should hold.*

Consider the following counterexample:

- *Let* $a = 1$, $b = 2$, *and* $c = 3$.
- $1 \leq 2^2 = 4$, *so* $(1, 2) \in R$.
- $2 \leq 3^2 = 9$, *so* $(2, 3) \in R$.
- *However,* $1 \leq 3^2 = 9$ *is true, so in this case, the transitivity holds.*

However, consider a different example:

- *Let* $a = 0.5$, $b = 1$, *and* $c = 0.5$.
- $0.5 \leq 1^2 = 1$, *so* $(0.5, 1) \in R$.
- $1 \leq (0.5)^2 = 0.25$ *is false, so* $(1, 0.5) \notin R$.

Thus, the relation is ***not transitive*** *in all cases.*

Conclusion:

The relation $R = \{(a, b) : a \leq b^2\}$ *is* ***neither reflexive, nor symmetric, nor transitive.***

3. Check whether the relation R defined in the set $\{1,2,3,4,5,6\}$ as $R = \{(a,b) : b = a+1\}$ is reflexive, symmetric, or transitive.

Solution :

- ***Reflexive***: *For reflexivity, we require that $a = b$. But in this relation, $b = a+1$, so no element is related to itself. Hence, R is **not reflexive**.*
- ***Symmetric***: *For symmetry, if $(a,b) \in R$, then (b,a) should also be in R. Since $b = a+1$, $a \neq b+1$, and thus R is **not symmetric**.*
- ***Transitive***: *For transitivity, if $(a,b) \in R$ and $(b,c) \in R$, then (a,c) should be in R. If $b = a+1$ and $c = b+1$, then $c = a+2$, which is not of the form $b = a+1$. Hence, R is **not transitive**.*

4. Show that the relation R in $\mathbf{R}$ (real numbers) defined as $R = \{(a,b) : a \leq b\}$ is reflexive and transitive but not symmetric.

Solution :

- ***Reflexive***: *For all $a \in \mathbf{R}$, $a \leq a$ holds, so R is **reflexive**.*
- ***Symmetric***: *If $a \leq b$, it is not necessarily true that $b \leq a$ (unless $a = b$). Hence, R is **not symmetric**.*
- ***Transitive***: *If $a \leq b$ and $b \leq c$, then $a \leq c$, so R is **transitive**.*

5. Check whether the relation R in $\mathbf{R}$ defined by $R = \{(a,b) : a \leq b^3\}$ is reflexive, symmetric, or transitive.

Solution :

- ***Reflexive***: *We need* $a \leq a^3$. *This is true for* $a = 1$, *but for* $0 < a < 1$, $a^3 < a$, *and for* $a < 0$, a^3 *may be greater than* a. *Thus,* R *is* ***not reflexive***.

- ***Symmetric***: *If* $a \leq b^3$, *it does not imply that* $b \leq a^3$. *Hence,* R *is* ***not symmetric***.

- ***Transitive***: *If* $a \leq b^3$ *and* $b \leq c^3$, *it does not necessarily imply that* $a \leq c^3$. *Thus,* R *is* ***not transitive***.

6. *Show that the relation* R *in the set* $\{1, 2, 3\}$ *given by* $R = \{(1, 2), (2, 1)\}$ *is symmetric but neither reflexive nor transitive.*

Solution :

- ***Reflexive***: *There are no pairs* $(1, 1), (2, 2), (3, 3)$ *in the relation, so* R *is* ***not reflexive***.

- ***Symmetric***: *Since* $(1, 2) \in R$ *implies that* $(2, 1) \in R$, *and vice versa,* R *is* ***symmetric***.

- ***Transitive***: *For transitivity, if* $(1, 2) \in R$ *and* $(2, 1) \in R$, *then we would need* $(1, 1) \in R$, *which is not true. Hence,* R *is* ***not transitive***.

7. *Show that the relation* R *in the set* A *of all the books in a library of a college, defined by* $R = \{(x, y) : x$ *and* y *have the same number of pages*$\}$, *is an equivalence relation.*

Solution :

- ***Reflexive***: *Every book has the same number of pages as itself, so R is* ***reflexive***.
- ***Symmetric***: *If book x has the same number of pages as book y, then book y has the same number of pages as book x. Hence, R is* ***symmetric***.
- ***Transitive***: *If book x has the same number of pages as book y, and book y has the same number of pages as book z, then book x has the same number of pages as book z. Thus, R is* ***transitive***.

Since R is reflexive, symmetric, and transitive, it is an ***equivalence relation***.

8. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation.

Solution :

- ***Reflexive***: *For all $a \in A$, $|a - a| = 0$, which is even. So, R is* ***reflexive***.
- ***Symmetric***: *If $|a - b|$ is even, then $|b - a|$ is also even (since $|a - b| = |b - a|$). So, R is* ***symmetric***.
- ***Transitive***: *If $|a - b|$ and $|b - c|$ are even, then $|a - c|$ is also even (since the sum of two even numbers is even). So, R is* ***transitive***.

Thus, R is an ***equivalence relation***.

9. Show that each of the relations R in the set $A = \{x \in \boldsymbol{Z} : 0 \leq x \leq 12\}$ given by:

(i) $R = \{(a,b) : |a-b| \text{ is a multiple of } 4\}$

Solution :

- ***Reflexive***: $|a-a| = 0$, *which is a multiple of 4, so R is* ***reflexive***.
- ***Symmetric***: *If* $|a-b|$ *is a multiple of 4, then* $|b-a|$ *is also a multiple of 4. So, R is* ***symmetric***.
- ***Transitive***: *If* $|a-b|$ *and* $|b-c|$ *are multiples of 4, then* $|a-c|$ *is also a multiple of 4. So, R is* ***transitive***.

(ii) $R = \{(a,b) : a = b\}$

Solution :

- *This is the identity relation, which is always* ***reflexive***, ***symmetric***, *and* ***transitive***.

10. Give an example of a relation that is:

(i) Symmetric but neither reflexive nor transitive.

Solution :

- *Example:* $R = \{(1,2),(2,1)\}$ *on the set* $\{1,2,3\}$.

(ii) Transitive but neither reflexive nor symmetric.

Solution :

- *Example:* $R = \{(1,2),(2,3),(1,3)\}$ *on the set* $\{1,2,3\}$.

(iii) Reflexive and symmetric but not transitive.

Solution :

- *Example:* $R = \{(1,1),(2,2),(1,2),(2,1)\}$ *on the set* $\{1,2\}$.

(iv) Reflexive and transitive but not symmetric.

Solution :

- *Example:* $R = \{(1,1),(2,2),(1,2)\}$ *on the set* $\{1,2\}$.

11. Show that the relation R *in the set* A *of points in a plane defined by* $R = \{(P,Q)$: *distance of* P *from the origin is the same as the distance of* $Q\}$ *is an equivalence relation. Further, show that the set of all points related to a point* $P \neq (0,0)$ *is the circle passing through* P *with origin as the center.*

Solution :

1. ***Reflexive****: The distance of any point* P *from the origin is always equal to its own distance. Therefore,* $(P,P) \in R$*, and the relation is* ***reflexive***.

2. ***Symmetric****: If the distance of* P *from the origin is equal to the distance of* Q *from the origin, then clearly the distance of* Q *from the origin is also equal to the distance of* P *from the origin. Thus,* R *is* ***symmetric***.

3. ***Transitive****: If* $(P,Q) \in R$ *and* $(Q,R) \in R$*, meaning that* P *and* Q *are equidistant from the origin and* Q *and* R *are equidistant from the origin, then* P *and* R *must also be equidistant from the origin. Hence,* $(P,R) \in R$*, and* R *is* ***transitive***.

Since the relation is reflexive, symmetric, and transitive, it is an ***equivalence relation***.

4. ***Circle passing through P****: The set of all points related to a point* $P \neq (0,0)$ *are those points that are equidistant from the origin.*

Geometrically, these points form a circle with the origin as the center and radius equal to the distance of P from the origin. Hence, the set of all points related to P forms the circle passing through P with the origin as the center.

12. *Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$ is an equivalence relation. Consider three right-angled triangles T_1, T_2, T_3 with sides 3, 4, 5; 5, 12, 13; and 6, 8, 10, respectively. Which triangles among T_1, T_2, T_3 are related ?*

Solution :

1. ***Reflexive**: Every triangle is similar to itself. Therefore, R is **reflexive**.*

2. ***Symmetric**: If triangle T_1 is similar to triangle T_2, then T_2 is also similar to T_1. Thus, R is **symmetric**.*

3. ***Transitive**: If triangle T_1 is similar to triangle T_2, and T_2 is similar to triangle T_3, then T_1 is similar to T_3. Therefore, R is **transitive**.*

*Since the relation is reflexive, symmetric, and transitive, it is an **equivalence relation**.*

4. ***Which triangles are related :***

- *The triangles $T_1 = \{3, 4, 5\}$ and $T_3 = \{6, 8, 10\}$ are similar because the sides of T_3 are proportional to those of T_1 (i.e., the sides of T_3 are twice the corresponding sides of T_1).*

- *However, $T_2 = \{5, 12, 13\}$ is not similar to either T_1 or T_3, as the* *side ratios are different.*

Thus, triangles T_1 and T_3 are related, but neither is related to T_2.

13. *Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1$ and P_2 have the same number of sides$\}$ is an equivalence relation. What is the set of all elements in A related to the right-angle triangle T with sides 3, 4, 5?*

Solution :

1. ***Reflexive***: *Every polygon has the same number of sides as itself. Therefore, R is **reflexive**.*

2. ***Symmetric***: *If polygon P_1 has the same number of sides as polygon P_2, then P_2 has the same number of sides as P_1. Thus, R is **symmetric**.*

3. ***Transitive***: *If polygon P_1 has the same number of sides as polygon P_2, and P_2 has the same number of sides as polygon P_3, then P_1 has the same number of sides as polygon P_3. Therefore, R is **transitive**.*

*Since the relation is reflexive, symmetric, and transitive, it is an **equivalence relation**.*

4. ***Polygons related to the right-angled triangle T:***

- *The set of all polygons related to the right-angled triangle T includes all **triangles** (since triangles are polygons with three sides).*

14. *Let L be the set of all lines in the XY-plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \parallel L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line*

$y = 2x + 4.$

Solution :

1. ***Reflexive***: *Every line is parallel to itself. Thus, R is* ***reflexive***.

2. ***Symmetric***: *If line* $L_1 \parallel L_2$, *then* $L_2 \parallel L_1$ *as well. Hence, R is* ***symmetric***.

3. ***Transitive***: *If* $L_1 \parallel L_2$ *and* $L_2 \parallel L_3$, *then* $L_1 \parallel L_3$. *Therefore, R is* ***transitive***.

Since R is reflexive, symmetric, and transitive, it is an ***equivalence relation***.

4. ***Lines related to*** $y = 2x + 4$***:***

- *All lines parallel to* $y = 2x + 4$ *will have the same slope of 2.*
- *Therefore, the set of all lines related to* $y = 2x + 4$ *includes all lines of the form* $y = 2x + c$, *where c is any constant.*

15. *Let R be the relation in the set* $\{1, 2, 3, 4\}$ *given by*
$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$.
Choose the correct answer.

(A) *R is reflexive and symmetric but not transitive.*

(B) *R is reflexive and transitive but not symmetric.*

(C) *R is symmetric and transitive but not reflexive.*

(D) *R is an equivalence relation.*

Solution :

- ***Reflexive**: The relation contains* $(1,1),(2,2),(3,3),(4,4)$, *so* R *is **reflexive**.*

- ***Symmetric**:* $(1,2) \in R$, *but* $(2,1) \notin R$, *so* R *is **not symmetric**.*

- ***Transitive**:* $(1,2) \in R$ *and* $(2,2) \in R$, *but* $(1,2) \in R$, *so it is **transitive**.*

*Thus, the correct answer is **(B)**: R is reflexive and transitive but not symmetric.*

16. Let R be the relation in the set N (natural numbers) given by $R = \{(a,b) : a = b-2 \text{ and } b > 6\}$. *Choose the correct answer.*

(A) $(2,4) \in R$

(B) $(3,8) \in R$

(C) $(6,8) \in R$

(D) $(7,7) \in R$

Solution :

We are given the relation $R = \{(a,b) : a = b-2 \text{ and } b > 6\}$, *which means that for a pair* (a,b) *to belong to* R, *the following two conditions must be satisfied:*

1. $a = b - 2$
2. $b > 6$

Let's check the options given one by one.

1. ***(A)*** $(2,4) \in R$**:**

- *Here* $a = 2$ *and* $b = 4$.

- *Check condition 1:* $2 = 4 - 2$, *which is true.*
- *heck condition 2:* $b = 4$, *but* 4 *is not greater than 6.*
- *Therefore,* ***(2, 4) is not in*** R.

2. ***(B)*** $(3, 8) \in R$**:**

- *Here* $a = 3$ *and* $b = 8$.
- *Check condition 1:* $3 = 8 - 2$, *which is false.*
- *Since the first condition is not satisfied,* $(3, 8)$ *is* ***not in*** R.

3. ***(C)*** $(6, 8) \in R$**:**

- *Here* $a = 6$ *and* $b = 8$.
- *Check condition 1:* $6 = 8 - 2$, *which is true.*
- *Check condition 2:* $b = 8$, *and* $8 > 6$, *which is also true.*
- *Therefore,* ***(6, 8) is in*** R.

4. ***(D)*** $(7, 7) \in R$**:**

- *Here* $a = 7$ *and* $b = 7$.
- *Check condition 1:* $7 = 7 - 2$, *which is false.*
- *Since the first condition is not satisfied,* $(7, 7)$ *is* ***not in*** R.

Correct answer: (C) $(6, 8) \in R$.